Final Exam

Due: Friday May 8, 2020 1:00 PM (Pacific Daylight Time)

Assignment description

Math 245 Final Exam Spring 2020

Please read all exam directions carefully

This exam is meant to take 2 hours to complete, although you are given an extra hour. It begins on Friday, May 8, at 10am, and is due at 1pm. Late exams will not be accepted -- plan things out to submit your solutions before the deadline, to account for unexpected internet outages, dead batteries, or whatnot. Some exam questions are similar to exercises from the textbook. You are expected to solve these questions directly, not by citing the textbook exercise as if it were a theorem you could use.

Cheating Policy:

You are permitted to use the following:

1. The textbook.
2. Any notes taken from lecture, including saved whiteboards and saved chats.
3. Any materials you personally created with your own hand previous to the exam, such as solutions to homework exercises.
4. Materials that someone you personally know created with their own hand previous to the exam and shared with you, such as class notes or homework solutions.
5. Any materials found on vadim.sdsu.edu, including old exams and solutions.
6. Paper, writing tools, a calculator.
7. A computer and/or a phone, but only if used to access the permitted materials above, and to submit your answers.
8. You may email the professor, Vadim Ponomarenko, (vponomarenko@sdsu.edu) for clarification of anything unclear on this exam.

You are NOT permitted to use any of the following:

1. Assistance from any other human being, in any fashion. The sole exception is correspondence with Vadim Ponomarenko about clarification of exam questions.
2. Any websites other than Crowdmark and vadim.sdsu.edu. Posting (or emailing) an exam question is cheating, whether or not help is provided.
3. Any books, papers, or materials, other than what is permitted above.

Even after you submit your exam solutions, do not discuss the exam with anyone until after 2pm on Friday, May 8. (several students get a little extra time)

Any violations of the above cheating policy will result in a report to SDSU's Center for Student Rights and Responsibilities, and will cause course failure or possibly worse.

If you witness a violation of the cheating policy and report it to the professor, you will be eligible for extra credit on the exam. All reports will be kept confidential.

Other Instructions:

Please write legibly, with plenty of white space. Fit your answers on blank paper, two or three answers per page, as specified in the instructions. If you wish, you may cut and rearrange pages. Be sure to write the number of each problem next to your solution. Be sure each image is in sharp focus, and oriented correctly, or points will be deducted. If necessary, take new images and reupload until you get it right. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. Some exam questions are similar to exercises from the textbook -- you are expected to solve these questions directly, not by citing the textbook exercise. Each problem is worth 5-10 points. You need not put your name on your answers, just on the pledge card.

Good luck!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Pledge (0 points)

Please print your name on a notecard (in the usual way, First LAST), and also sign your legal name in the middle of the card. Exams without this signed pledge will earn a score of 0.

Your signature means that you agree to the following pledge:

You swear or affirm that (1) You have carefully read the cheating policy above; and (2) You will personally follow, to the best of your ability, the cheating policy above; and (3) You will not assist anyone else in violating the cheating policy above; and (4) If you violate the cheating policy, or help someone else to do so, you hope that you get caught and punished.

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Page 1 (30 points)

Write solutions to the following three questions on one 8.5x11 page. Photograph that page, and upload it. Be sure to orient that image correctly, and that it is in sharp focus. Put your answers in consecutive order, i.e. #1 first, then #2, then #3. You need not write out the question, only the question number.

1. Find a tautology using basic propositions p, q. Your solution must use each of p, q at least once, and may not use special propositions T or F. Justify your answer with a truth table.
2. Find a predicate with one free variable, x (with domain $\mathbb{Z}$), such that the predicate is true for exactly two choices for x . Please put a box around your predicate, and justify your answer.
3. Let $A = \{a, b\}$ and $B = \{x\}$. Find sets S, T such that $S\Delta T = A \times B$ and $S \cap T = B \times A$. Give S, T in list notation.

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Page 2 (30 points)

Write solutions to the following three questions on one 8.5x11 page. Photograph that page, and upload it. Be sure to orient that image correctly, and that it is in sharp focus. Put your answers in consecutive order, i.e. #4 first, then #5, then #6. You need not write out the question, only the question number.

4. Let S denote the set of letters in your first name. (e.g. if your name is Todd then $S = \{t, o, d\}$). Find a relation on S that is left-total and not right-definite. Give S and your relation in list notation.
5. Let S again denote the set of letters in your first name. Give one element from $S \times 2^S$ (your choice), and calculate $|S \times 2^S|$. Be sure to use proper notation.
6. Prove that $n^{17} \neq O(n^3)$, using the definition of O. (You may not use Thm. 7.10).

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Page 3 (30 points)

Write solutions to the following three questions on one 8.5x11 page. Photograph that page, and upload it. Be sure to orient that image correctly, and that it is in sharp focus. Put your answers in consecutive order, i.e. #7 first, then #8, then #9. You need not write out the question, only the question number.

7. Define relation R on $\mathbb{Q}$ via $R = \{(a, b) : 2a - 2b \in \mathbb{Z}\}$. Prove or disprove that R is an equivalence relation. Hint: compare with exercise 11.1.
8. Use the method developed in exercises 11.6, 11.7, 11.8, to find an integer $z \in [0, 11)$ such that $z \equiv 3^{96} \pmod{11}$. Do not actually compute 3^{96}, and explain each step of your calculation.
9. Let $\equiv$ denote the equivalence relation modulo 7, on $\mathbb{Z}$. Define $[a] - [b] = \{x - y : x \in [a], y \in [b]\}$, a subset of $\mathbb{Z}$. Prove that, for all $a, b \in \mathbb{Z}$, that $[a] - [b] = [a - b]$ (equal as sets). Hint: compare with exercise 11.19.

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Page 4 (30 points)

Write solutions to the following three questions on one 8.5x11 page. Photograph that page, and upload it. Be sure to orient that image correctly, and that it is in sharp focus. Put your answers in consecutive order, i.e. #10 first, then #11, then #12. You need not write out the question, only the question number.

For exercises 10,11, consider the divisibility relation $|$ on the set $S = \{2, 3, 4, 5, 6, 10, 12, 15, 20\}$. This can be thought of as the poset $(|, \mathbb{N})$ restricted to S.

10. Draw the Hasse diagram for $|$ on S, as defined above.
11. Find the height and width of the poset $(|, S)$, as defined above. Justify your answer.
12. Let S be a set, R be a partial order on S, and $T \subseteq S$. Let $U \subseteq S$ be the set of all upper bounds of T. Let $G \subseteq S$ be the set of all greatest elements of T. Prove that $G = U \cap T$. Hint: compare with exercise 12.12.

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Page 5 (30 points)

Write solutions to the following three questions on one 8.5x11 page. Photograph that page, and upload it. Be sure to orient that image correctly, and that it is in sharp focus. Put your answers in consecutive order, i.e. #13 first, then #14, then #15. You need not write out the question, only the question number.

13. Let S, T be sets, and R a relation from S to T. Prove that R is right-total if and only if R^{-1} is left-total. Hint: compare with exercise 13.4.
14. Consider the function $f : \mathbb{N} \to \mathbb{N}$ given by $f(n) = \frac{n(n+1)}{2}$. Prove that f is injective. Hint: compare with exercise 13.14.
15. Consider F_1, F_2, functions on $\mathbb{R}$, given by $F_1(x) = e^x$ and $F_2(x) = \begin{cases} \ln x & x > 0 \\ 5 & x \leq 0 \end{cases}$. Prove that $F_2 \circ F_1$ and F_2 are surjective. Hint: compare with exercise 13.22.

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Page 6 (30 points)

Write solutions to the following three questions on one 8.5x11 page. Photograph that page, and upload it. Be sure to orient that image correctly, and that it is in sharp focus. Put your answers in consecutive order, i.e. #16 first, then #17, then #18. You need not write out the question, only the question number.

All of the remaining questions (16,17,18,19,20) concern $\mathbb{S}$, defined as the set of all functions from $\mathbb{Z}$ to $\mathbb{Z}$, i.e.

$$\mathbb{S} = \{f | f : \mathbb{Z} \to \mathbb{Z}\} = \{f \subseteq \mathbb{Z} \times \mathbb{Z} | f \text{ is left-total and right-definite}\}.$$

16. Let $f = \{(x, y) : y = x^3\}$, a relation on $\mathbb{Z}$. Prove that $f \in \mathbb{S}$.
17. Consider the relation R_1 on $\mathbb{S}$ given by
$$R_1 = \{(f, g) : \forall x \in \mathbb{Z}, 0 \leq f(x) \leq g(x)\}.$$
Prove that R_1 is antisymmetric.
18. Consider again the relation R_1 on $\mathbb{S}$ given by
$$R_1 = \{(f, g) : \forall x \in \mathbb{Z}, 0 \leq f(x) \leq g(x)\}.$$
Prove or disprove that R_1 is a partial order.

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Page 7 (20 points)

Write solutions to the following two questions on one 8.5x11 page. Photograph that page, and upload it. Be sure to orient that image correctly, and that it is in sharp focus. Put your answers in consecutive order, i.e. #19 first, then #20. You need not write out the question, only the question number.

Recall that $\mathbb{S}$ is the set of all functions from $\mathbb{Z}$ to $\mathbb{Z}$, as defined on the previous page. For questions 19,20, we define relation R_2 on $\mathbb{S}$ given by

$$R_2 = \{(f, g) : \forall x \in \mathbb{Z}, f(x) = g(-x)\}.$$

19. Prove or disprove that R_2 is an equivalence relation.
20. Prove or disprove that R_2 is a function.

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Before you submit, please ensure that your pages are in order and rotated correctly.
You will not be able to resubmit your work after the due date has passed.

Submit for evaluation